differentiation

1. Functions f and g are given by $f(x) = 3x + 1$ and $g(x) = x^2 - 2$.

    (a) (i) Find $p(x)$ where $p(x) = f(g(x))$.

        (ii) Find $q(x)$ where $q(x) = g(f(x))$.   **3**

    (b) Solve $p'(x) = q'(x)$.   **3**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(a)	3	C	CN	A4	$3(x^2 - 2) + 1, (3x + 1)^2 - 2$	2009 P2 Q2
(b)	3	C	CN	C1	$x = -\frac{1}{2}$	

$\bullet^1$ ss: substitute for g(x) in f (x)	$\bullet^1$ $f(x^2 - 2)$
$\bullet^2$ ic: complete	$\bullet^2$ $3(x^2 - 2) + 1$
$\bullet^3$ ic: sub. and complete for q(x)	$\bullet^3$ $(3x + 1)^2 - 2$
$\bullet^4$ ss: simplify	$\bullet^4$ $p(x) = 3x^2 - 5, q(x) = 9x^2 + 6x - 1$
$\bullet^5$ pd: differentiate	$\bullet^5$ $p'(x) = 6x, q'(x) = 18x + 6$
$\bullet^6$ pd: solve	$\bullet^6$ $x = -\frac{1}{2}$

2. If $y = x^2 - x$, show that $\dfrac{dy}{dx} = 1 + \dfrac{2y}{x}$.   **3**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	1	C	NC	C1		1989 P1 Q12
	2	A/B	NC	A6, CGD		

$\bullet^1$ $\dfrac{dy}{dx} = 2x - 1$

$\bullet^2$ $RHS = 1 + \dfrac{2\left(x^2 - x\right)}{x}$

$\bullet^3$ $1 + 2(x - 1)$ and complete

3. Given $f(x) = 3x^2(2x - 1)$, find $f'(-1)$.   **3**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	3	C	NC	C1		1999 P1 Q5

$\bullet^1$ $6x^3 - 3x^2$

$\bullet^2$ $18x^2 - 6x$

$\bullet^3$ 24

4. Find $\dfrac{dy}{dx}$ where $y = \dfrac{4}{x^2} + x\sqrt{x}$. **4**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	4	C	NC	C1		1995 P1 Q7

- $\bullet^1$ $4x^{-2}$ stated or implied by $\bullet^3$
- $\bullet^2$ $+x^{\frac{3}{2}}$ stated or implied by $\bullet^4$
- $\bullet^3$ $-8x^{-3}$
- $\bullet^4$ $+\frac{3}{2}x^{\frac{1}{2}}$

5. Find $f'(4)$ where $f(x) = \dfrac{x-1}{\sqrt{x}}$. **5**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	5	C	NC	C1		1996 P1 Q9

- $\bullet^1$ $\frac{x}{\sqrt{x}} - \frac{1}{\sqrt{x}}$ or $x \times x^{-\frac{1}{2}} - 1 \times x^{-\frac{1}{2}}$
- $\bullet^2$ $x^{\frac{1}{2}} - x^{-\frac{1}{2}}$
- $\bullet^3$ $\frac{1}{2}x^{-\frac{1}{2}}$
- $\bullet^4$ $\frac{1}{2}x^{-\frac{3}{2}}$
- $\bullet^5$ $\frac{5}{16}$

6. Given that $y = 2x^2 + x$, find $\dfrac{dy}{dx}$ and hence show that $x\left(1 + \dfrac{dy}{dx}\right) = 2y$. **3**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	3	C	NC	C1		1997 P1 Q8

- $\bullet^1$ $\frac{dy}{dx} = 4x + 1$
- $\bullet^2$ $LHS = x(1 + 4x + 1)$ or $RHS = 2\left(2x^2 + x\right)$
- $\bullet^3$ completes proof

7. Differentiate $2\sqrt{x}(x + 2)$ with respect to x. **4**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	4	C	NC	C1		1998 P1 Q14

- $\bullet^1$ know to expand
- $\bullet^2$ $2x^{\frac{3}{2}} + 4x^{\frac{1}{2}}$
- $\bullet^3$ $3x^{\frac{1}{2}}$
- $\bullet^4$ $2x^{-\frac{1}{2}}$

8. If $f(x) = kx^3 + 5x - 1$ and $f'(1) = 14$, find the value of k. **3**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	3	C	NC	C1, A6		1994 P1 Q2

- $\bullet^1$ $f'(x) = 3kx^2 + 5$
- $\bullet^2$ $f'(1) = 3k + 5$
- $\bullet^3$ $k = 3$

9. If $f(x) = \cos^2 x - \dfrac{2}{3x^2}$, find $f'(x)$. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	2	C	NC	C21, C1		1990 P1 Q19
	2	A/B	NC	C21, C1		

- $\bullet^1$ $-\frac{2}{3}x^{-2}$
- $\bullet^2$ $2\cos x$
- $\bullet^3$ $\times(-\sin x)$
- $\bullet^4$ $\frac{4}{3}x^{-3}$

10. Differentiate $4\sqrt{x} + 3\cos 2x$ with respect to x. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	2	C	NC	C21, C1		1993 P1 Q9
	2	A/B	NC	C21, C1		

- $\bullet^1$ $4x^{\frac{1}{2}}$
- $\bullet^2$ $2x^{-\frac{1}{2}}$
- $\bullet^3$ $-\sin 2x$
- $\bullet^4$ $\times 2$

11. Differentiate $\sin 2x + \dfrac{2}{\sqrt{x}}$ with respect to x. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	2	C	NC	C3		1989 P1 Q10
	2	A/B	NC	C20		

- $\bullet^1$ $2x^{-\frac{1}{2}}$
- $\bullet^2$ $\cos 2x$
- $\bullet^3$ $\times 2$
- $\bullet^4$ $-x^{-\frac{3}{2}}$

[END OF QUESTIONS]